Welcome to AP Pre-Calculus!!

You are currently enrolled in AP Pre-Calculus at Barnegat High School for the 2023-2024 school year. Attached is a packet containing items we feel are necessary for you to have mastered without the use of a calculator to do well in AP Pre-Calculus.

In order to begin our journey, you must do some "mathercise" to keep your mind in shape through the summer months. So, I have prepared the attached assignment to help you "burn" your mathematical muscles! Make sure all of your work is presented in a logical order showing ALL of your work indicating your answers clearly. IF I CAN'T READ YOUR HANDWRITING, IT IS WRONG!! This packet is a review of some Algebra II topics; it is to be done NEATLY and on a SEPARATE sheet of paper.

This packet must be completed by September 6, 2023 and it will be graded. We will go over all questions on the material in the packet for the first two or three days and then you will be tested on this material.

Enjoy your summer and we look forward to meeting you in September!
Sincerely,

Mrs. Barbara Quick

P.S.: You **MUST** be familiar with the graphs of $y = x, y = x^2, y = x^3, y = \sqrt{x}, y = e^x, y = \ln x, y = \frac{1}{x}$.

There will be a quiz on day 2 of class.

AP Pre-Calculus
Summer Assignment
All Work MUST be shown on a separate sheet of paper!!

1. Simplifying Radicals

Radicals

To simplify means that no radicand has a perfect square factor and there is no radical in the denominator

Product Property: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example:

Simplify $\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$

Simplify $\sqrt{\frac{15}{2}} = \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{30}}{\sqrt{4}} = \frac{\sqrt{30}}{2}$

a) $\sqrt{32}$

b) $\sqrt{(2x)^8}$

c) $\sqrt{49m^2n^8}$

d) $\sqrt[3]{-64}$

e) $\sqrt{\frac{11}{25}}$

f) $\sqrt{60} \cdot \sqrt{105}$

g) $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{6})$

h) $\frac{3}{2-\sqrt{5}}$

i) $\sqrt{8x^3y^5} \cdot \sqrt{5x^7y^9}$

2. Rules of Exponents

In pre-calculus, you will be required to perform algebraic manipulations with negative exponents as well as fractional exponents. By definition: $x^{-n} = \frac{1}{x^n}$ and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

As a reminder, rules of exponents are as follows:

When we multiply, we add exponents $x^a \cdot x^b = x^{a+b}$

When we divide, we subtract exponents $\frac{x^a}{x^b} = x^{a-b}$, $x \neq 0$

When we raise powers, we multiply $\left(x^a\right)^b = x^{a \cdot b}$

Simplify and write with positive exponents.

1. $-8x^{-2}$

$\frac{-8}{x^2}$

2. $\left(-5x^3\right)^{-2}$

$(-5)^{-2}x^{3-2} = \frac{1}{(-5)^2 x^6} = \frac{1}{25x^6}$

3. $\left(-\frac{3}{x^4}\right)^{-2}$

$\frac{(-3)^{-2}}{\left(x^4\right)^{-2}} = \frac{1}{(-3)^2 x^{-8}} = \frac{x^8}{9}$

4. $\left(36x^{10}\right)^{1/2}$

$\sqrt{36x^{10}} = 6x^5$

5. $\left(27x^3\right)^{-2/3}$

$\frac{1}{\left(27x^3\right)^{2/3}} = \frac{1}{\left(\sqrt[3]{27x^3}\right)^2} = \frac{1}{9x^2}$

6. $\left(16x^{-2}\right)^{3/4}$

$16^{3/4}x^{-6/4} = \left(\sqrt[4]{16}\right)^3 \frac{1}{x^{3/2}} = \frac{8}{x^3/2}$

Simplify each expression (write with no negative exponents):

a) $5x^2 \bullet 2x^5$

b) $\left(4xyz\right)^2\left(3x^2y^{-2}z\right)^{-3}$

c) $\left(x^m\right)^n\left(x^n\right)^{n-m}$

d) $\frac{(2x^2)^3y^2}{x^3y^4}$

e) $\frac{\left(x^{-3}y^2\right)^{-4}}{\left(y^6x^{-4}\right)^{-2}}$

f) $\left(\frac{4a^3b}{a^2b^3}\right)\left(\frac{3b^2}{2a^2b^4}\right)$

g) $\left(8x^6\right)^{-4/3}$

h) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right)$

3. Factor completely

Special Forms

Common factor: $x^3 + x^2 + x = x(x^2 + x + 1)$

Difference of squares: $x^2 - y^2 = (x + y)(x - y)$

Perfect squares: $x^2 + 2xy + y^2 = (x + y)^2$

Perfect squares: $x^2 - 2xy + y^2 = (x - y)^2$

Sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ - Trinomial unfactorable

Difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ Trinomial unfactorable

Grouping: $xy + xb + ay + ab = x(y + b) + a(y + b) = (y + b)(x + a)$

a) $6x^2 + 7x - 5$ b) $4x^5 - 16x^3$ c) $x^3 - 8$ d) $x^3 - 2x^2 - 4x + 8$

e) $5x^2y - 30xy^2 + 40y^3$ f) $y^4 - y^2 - 72$ g) $(3x - 2)^2 - 2(3x - 2) - 35$ h) $-20x^4 + 5x^2$

4. Solving Linear Equations and Inequalities

a) $6(5x + 1) - 2(x - 2) = 5x + 10$ b) $\frac{x-2}{3} + \frac{x+5}{2} = \frac{1}{3}$ c) $5(x + 3) + 4 < x - 1$

d) $2 \le 4 - 3x < 7$ e) $\frac{y+2}{3} - \frac{y+1}{6} > 1$ f) $x^2 + 3x - -4 = 14$

g) $3\sqrt{x - 2} - 8 = 8$ h) $12x^2 = 3x$ i) $|3 - 2x| + 2 > 5$

j) $|4x + 1| = 3$ k) $x^3 - x^2 - 30x \le 0$ l) $\left|\frac{x+7}{5}\right| > 2$

5. Operations with Imaginary Numbers

a) $(3+5i)+(12-7i)$

b) $(2-8i)-(-9-i)$

c) $(1-i)(5+2i)$

d) $(1+2i)^2$

e) $\frac{1+6i}{5i}$

6. Linear Functions

Remember, in order to write the equation of a line, you need the slope of the line and a point.

Slope between two points (x_1, y_1) and $:(x_2, y_2)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

a) Write the equation of the line through the points (-1, 16) and (4, 2)

b) Write the equation of the line through (1, 3) and parallel to $2x + 3y = -5$

c) Write the equation of the line through the point (3, 2) with slope $m = \frac{4}{5}$

d) Write the equation of the line through (2, 4) with $m = 0$

e) Write the equation of a line with $f(2) = -5$ and $f(-3) = 1$

f) Write the equation of a line perpendicular to $-3y + 6x = 2$ through (4, 3)

g) Write the equation of a line with x-intercept of 3 and a y-intercept of 3.

h) The table below represents a linear function. Fill in the missing values.

x	-2		4	7		11
y	10	1		-17	-26	-29

7. Functions

Evaluate the function for the given value.

$f(x) = x^2 + 5$ $\quad g(x) = \sqrt{x-3}$ $\quad k(x) = 7x - 9$

a) $f(-4) =$

b) $f(9a) =$

c) $f(x+1) =$

d) $g(0) =$

e) $g(-6) =$

f) $f(x+h) - f(x) =$

g) $\frac{k(x+h)-k(x)}{h}$

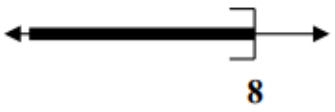
h) $g(f(x)) =$

i) $k^{-1}(x) =$

j) $f(f(2)) =$

8. Interval Notation

Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

9. Solving Systems of Equations

$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$

Substitution

- Solve 1 equation for 1 variable
- Rearrange
- Plug into the other equation and solve.
- Solve the equation.

$3x + y = 6$
$y = 6 - 3x$
$2x - 2(6 - 3x) = 4$
$2x - 12 + 6x = 4$
$8x - 12 = 4$
$8x = 16$
$x = 2$

$y = 6 - 3(2) = 6 - 6 = 0$

Solution: $(2, 0)$

Elimination

- Find the opposite coefficient for 1 variable.
- Multiply equation(s) by constant
- Add equations together, eliminating one variable
- Solve the equation.

$2(3x + y = 6)$

$6x + 2y = 12$
$2x - 2y = 4$

$8x = 16$
$x = 2$

Plug x into either equation and solve for y
$2(2) - 2y = 4 \rightarrow -2y = 0 \rightarrow y = 0$

Solution: $(2, 0)$

Solve each system

a) $\begin{cases} x + 2y = 1 \\ 5x - 4y = -23 \end{cases}$

b) $\begin{cases} -8x - 10y = 24 \\ 6x + 5y = 2 \end{cases}$

c) $\begin{cases} -2x + 7y = 4 \\ -5x - 9y = 10 \end{cases}$

d) $\begin{cases} y = x^2 - 2x - 3 \\ y = 2x - 3 \end{cases}$

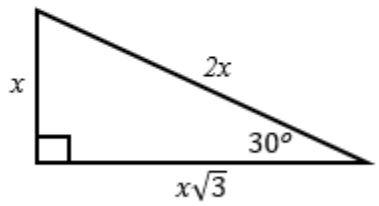
e) $\begin{cases} y = (x + 2)^2 - 6 \\ y = 4x - 2 \end{cases}$

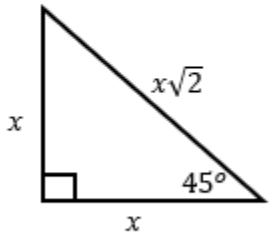
f) $\begin{cases} x^2 + y^2 = 25 \\ 2x + y = 10 \end{cases}$

10. Right Triangles

Pythagorean Theorem: $a^2 + b^2 = c^2$

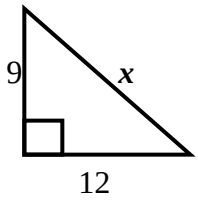
Special Right Triangles





Find the value of x.

a)



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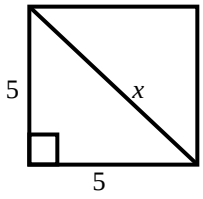

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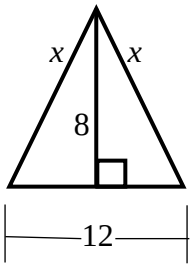
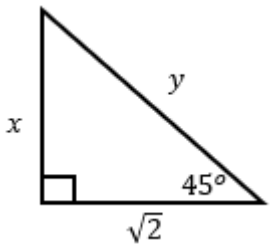


Solve for x and y.

d)



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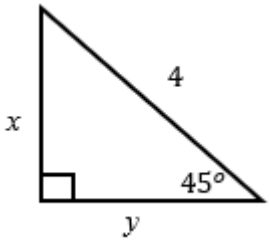
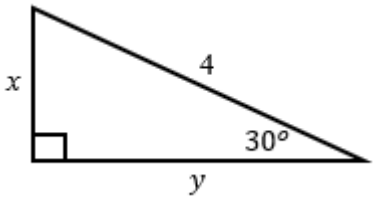
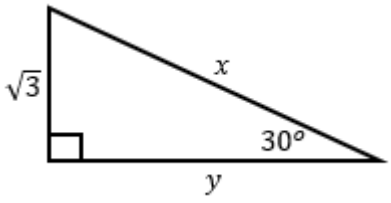


f)



g)



11. Rational Expressions

Adding & Subtracting

Find the least common denominator. Write each fraction with the LCD. Add/subtract the numerators as indicated and leave the denominators as they are.

$$\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}=\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}=$$

$$\frac{2}{2}\cdot\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}\cdot\frac{x}{x}=\frac{6x+2+5x^2-4x}{2x(x+2)}=$$

$$\frac{5x^2+2x+2}{2x(x+2)}$$

Multiplying & Dividing

Factor the numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, flip the second fraction and multiply.

$$\frac{x^2+10x+21}{5-4x-x^2}\cdot\frac{x^2+2x-15}{x^3+4x^2-21x}$$

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)}\cdot\frac{(x+5)(x-3)}{x(x-3)(x+7)}$$

$$=\frac{(x+3)}{x(1-x)}$$

Complex Fractions

Multiply the top and bottom of the fraction by the common denominator and then simplify.

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\left(\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\right)\cdot\left(\frac{x+1}{x+1}\right)=$$

$$\frac{-7(x+1)-\frac{6(x+1)}{x+1}}{\frac{5(x+1)}{x+1}}=\frac{-7x-7-6}{5}=$$

$$\frac{-7x-13}{5}$$

Simplify each expression

a) $\dfrac{3x+18}{x^2+6x}$

b) $\dfrac{x^2-5x+6}{x+4}\cdot\dfrac{3x+12}{x-2}$

c) $\dfrac{6x-9}{5x+1}\div\dfrac{6-13x+6x^2}{15x^2-7x-2}$

d) $\dfrac{y-x}{x^2y}+\dfrac{x+y}{xy^2}$

e) $\dfrac{2-x^2}{x^2+x}+\dfrac{3x+4}{3x+3}$

f) $\dfrac{2x+3}{5x-30}-\dfrac{3x+4}{x-6}$

g) $\dfrac{1+\frac{1}{x}}{x+1}$

h) $\dfrac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}$

i) $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}$

12. Word Problems

a) In 1998, there were 47 million people worldwide who had been living with HIV. At that time, the infection rate was 5.8 million people per year. (Source: United Nations AIDS and World Health Organization.)

- Write a formula for a linear function f that models the total number of people in millions who were living with HIV x years after 1998.
- Estimate the number of people who may have been living with HIV by the year 2006.

b) By noon, 3 inches of rain had fallen during a storm. Rain continued to fall at a rate of ¼ inch per hour.

- Find a formula for a linear function f that models the total amount of rainfall x hours past noon.
- Find the total amount of rainfall by 2:30 p.m.

c) A large pizza at Palanzio's Pizzeria costs $6.80 plus $0.90 for each topping. The cost of a large cheese pizza at Guido's Pizza is $7.30 plus $0.65 for each topping. How many toppings need to be added to a large cheese pizza from Palanzio's Pizzeria and Guido's Pizza in order for the pizzas to cost the same, not including tax?

d) Tickets to a concert are $5 for balcony seats and $10 for orchestra seats. If attendance was 600 and total receipts were $4750, how many people bought orchestra seats?

e) A water tank has the shape of a cone. The tank is 10 m high and has a radius of 3 m at the top. If the water is 5 m deep (in the middle) what is the surface area of the top of the water?

f) Two cars start moving from the same point. One travels south at 70 mi/hr, the other west at 55 mi/hr. How far apart are they two hours later?

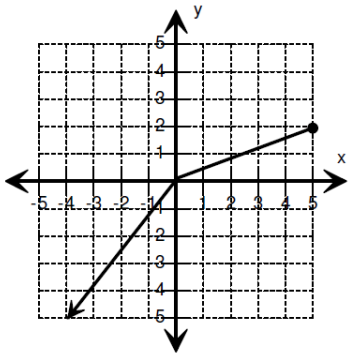
g) Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount yearly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed $90, while one using 1600 cubic feet was billed $105.
   - What is the charge per cubic foot?
   - Write an equation for the total cost of a resident's water as a function of cubic feet of water used.
   - How many cubic feet of water used would lead to a bill of $130?

h) The table gives the average weight, w, in pounds of American men in their sixties for various heights, h, in inches.
   - How do you know that the data in this table could represent a linear function?
   - Find weight, w, as a linear function of height, h. What is the slope of the line? What are the units for the slope?
   - Find height as a linear function of weight. What is the slope of the line? What are the units for the slope?

h(inches)	68	69	70	71	72	73	74	75
w (pounds)	166	171	176	181	186	191	196	201

13. Graphs: For each graph pictured below, state the…
- Domain
- Range
- x-intercepts
- y-intercept

a.



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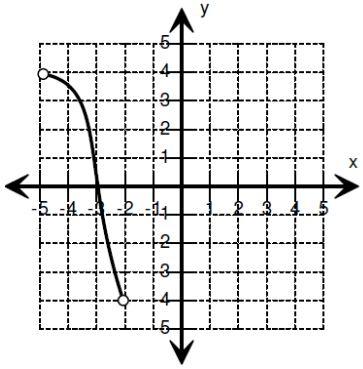
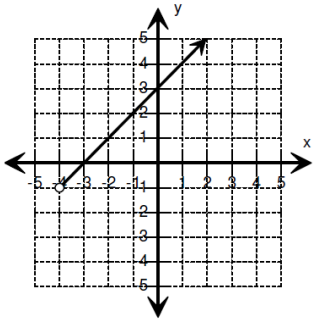


c.



d.